

A Possible Mechanism for Production of Primordial Black Holes

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Abstract

Primordial Black Hole Remnants(PBHRs) can be considered as a primary source of cold dark matter. Hybrid inflation provides a possible framework for production of primordial black holes(PBHs) and these PBHs evaporate subsequently to produce PBHRs. In this paper we provide another framework for production of these PBHs. Using signature changing cosmological model and the generalized uncertainty principle as our primary inputs, first we find a geometric cosmological constant for early stage of universe evolution. This geometric cosmological constant can lead to heavy vacuum density which may be interpreted as a source of PBHs production during the inflationary phase. In the next step, since it is possible in general to have non-vanishing energy-momentum tensor for signature changing hypersurface, this non-vanishing energy-momentum tensor can be considered as a source of PBHs production. These PBHs then evaporate via the Hawking process to produce PBHRs. Finally, possible observational schemes for detecting relics of these PBHRs are discussed.

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Key Words: Black Hole Physics, Dark Matter, Quantum Cosmology, Hartle-Hawking Proposal, Generalized Uncertainty Principle

1 Motivation

Dark matter constitutes a great part of the matter in the universe. There are several candidates for dark matter[1]. Recently, black holes remnants have been considered as a possible source of cold dark matter[2,3]. In fact, Generalized Uncertainty Principle(GUP) prevents black holes from total evaporation and the ultimate phase of an evaporating black hole is a Planck size remnants. Evaporating Primordial black holes provide possible framework for such Planck size remnants[4]. Possible creation of black hole remnants in inflationary scenarios, ultrahigh energy cosmic ray (UHECR) airshowers and also in Large Hadronic Collider(LHC) have been investigated[5-7]. Here we are going to provide another model for creation of evaporating primordial black holes based on quantum cosmological considerations. The possible production of PBHRs in Hartle-Hawking quantum cosmology first has been emphasized by Chen and Adler[4]. However, they have not provided a thorough formulation of their conjecture. Here we are going to provide a framework for possible production of PBHs in Hartle-Hawking scenario of quantum cosmology. Our calculation is based on ref.[8]. First we will construct a signature changing cosmological model. Then a geometrical interpretation of the cosmological constant will be given based on the generalized uncertainty principle. This cosmological constant can be considered as a heavy vacuum density which may be possible source of PBHs production. Finally we will show that non-vanishing energy-momentum tensor of change surface may also be a possible source of PBHs production. The essence of our analysis is the fact that quantum fluctuations of spacetime geometry in Planck scale may produce non-vanishing cosmological constant and non-vanishing energy-momentum tensor(due to non-vanishing jump of the extrinsic curvature) and these entities can be considered as possible sources of primordial black hole production. We use the thin shell formalism of general relativity within Colombeau new theory of generalized functions. We also use the generalized uncertainty principle(GUP), existence of minimal length scale and the Hartle-Hawking no-boundary proposal as our primary inputs.

The structure of the paper is as follows: section 2 is devoted to provide our model based on signature change and the generalized uncertainty principle. In this section we will find a cosmological constant with geometric origin which can lead to heavy vacuum density deriving inflation. The possible non-vanishing of energy-momentum tensor of signature change surface due to non-vanishing jump of the extrinsic curvature is described as a source of PBHs production. In section 3, following the arguments of Chen and Adler[4],

we investigate the possible production of PBHRs in the signature change framework. Experimental and observational schemes for detection of PBHRs are discussed in section 4. The paper follows by summary and conclusions in section 5.

2 Signature Change, GUP and the Cosmological Constant

The idea of signature change has been originated from the Hartle-Hawking No-Boundary proposal[9]. The essence of the idea of signature change can be summarized as follows: The universe was initially Riemannian 4-sphere with Euclidean signature $(+, +, +, +)$ of metric, and then by a change of signature of spacetime metric(via a quantum tunneling process), the transition to usual Lorentzian(de Sitter) spacetime with metric signature of $(-, +, +, +)$ have been occurred. The transition had been occurred on a hypersurface with 3-sphere topology.

Earlier attempts to describe this proposal was based on Euclidean path integral formulation of quantum gravity and the analogy to quantum tunneling effect in quantum mechanics[10], but it has been revealed that the idea of signature change can be formulated in the language of classical general relativity[11]. Among these literature, Mansouri and Nozari have constructed a cosmological model with signature change in the framework of Colombeau algebra[8,12]. In which follows, we use Mansouri-Nozari results to provide our model of PBHs production.

2.1 A Geometric Cosmological Constant

According to the Hartle-Hawking no-boundary proposal, the universe after signature change should be a de Sitter universe (inflationary phase). Let us assume that the space time after signature change is a de Sitter one. Consider now the following de Sitter metric with appropriate lapse function $f(t)$ in order to produce signature change at $t = 0$. The $t = \text{const.}$ sections of this metric are surfaces of constant curvature $k = 1$ [13]

$$ds^2 = -f(t)dt^2 + a^2(t)\left(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)\right) \quad (1)$$

where $f(t)$ and $a(t)$ are defined as

$$f(t) = \theta(t) - \theta(-t) \quad (2)$$

and

$$a^2(t) = \alpha_+^2 \cosh^2(\alpha_+^{-1}t)\theta(t) + \alpha_-^2 \cos^2(\alpha_-^{-1}t)\theta(-t). \quad (3)$$

Suppose that $[a] = 0$, therefore we will have $\alpha_+ = \alpha_- := R$, where R is the radius of 3-surface of signature change. Now, the Euclidean sector can be interpreted as a S^4 with S^3 sections defined by $t = \text{const}$. The boundary of the Euclidean sector, defined by $t = 0$, is a S^3 having the radius $R = \alpha_- = H_0^{-1}$ which is the maximum value of $\alpha_- \cos(\alpha_-^{-1}t)$ (see Fig. 1). In the Lorentzian sector the cosmological constant is given by $\Lambda = 3\alpha_+^{-2} = 3H_0^2$. The $t = \text{const}$. surfaces are S^3 with radius $\alpha_+ \cosh(\alpha_+^{-1}t)$ having the minimum value $R = \alpha_+ = H_0^{-1}$ (see Fig.1). Therefore, the following relation between the cosmological constant and the radius of the boundary is obtained:

$$\Lambda = \frac{3}{R^2}. \quad (4)$$

According to the string theory and loop quantum gravity, there is a minimum length scale of the order of Planck length which restricts the accuracy of measurement of distances[14]. In other words, one of the most interesting consequences of unification of gravity and quantum mechanics is that in resulting quantum gravity scenario, there exists a minimal observable distance on the order of the Planck length, $l_P = \sqrt{\frac{G\hbar}{c^3}} \sim 10^{-33}cm$, where G is the Newton constant. The existence of such a fundamental length is a dynamical phenomenon due to the fact that, at Planck scale, there are fluctuations of the background metric, i.e. a limit of the order of Planck length appears when quantum fluctuations of the gravitational field are taken into account. In the language of string theory one can say that a string cannot probe distances smaller than its length. The noncommutativity of spacetime at quantum gravity level supports this idea[15]. This minimal length scale can be obtained from the following generalized uncertainty principle

$$\Delta x \Delta p \geq \frac{\hbar}{2}(1 + \beta(\Delta p)^2) \quad (5)$$

which yields $(\Delta x)_{\min} = \hbar\sqrt{\beta}$ and this is on the order of Planck length [16]. Now it is reasonable to set $R \sim l_P$ and therefore

$$\Lambda \sim \frac{3}{l_P^2}. \quad (6)$$

This statement shows the possibility of having an exponentially expanding universe with heavy vacuum density probably equal to the Planck density. This heavy vacuum density can be considered as a source of PBHs production during inflationary era[17].

2.2 Non-Vanishing Energy-Momentum Tensor of Signature Change Hypersurface

Following the formalism of shell dynamics in general relativity and using Colombeau algebra [18], one can show that elements of energy-momentum tensor of the signature change hypersurface are

$$\kappa S_\mu^\nu = \text{diag}(0, \Upsilon, \Upsilon, \Upsilon), \quad (7)$$

where Υ is defined as

$$\Upsilon = \left(\frac{2}{R} \tanh(R^{-1}t) - \frac{2}{R} \tan(R^{-1}t) \right) |_{(t=0)} (\tau - 1) \quad (8)$$

where $\frac{1}{2} < \tau < 1$ is a constant [8]. We therefore conclude that given the de Sitter metric in the form of (1) the energy-momentum tensor of the hypersurface of signature change defined by $t = 0$ vanishes. However, this is not a general statement since one could require a matching along other sections corresponding to a non-maximum radius of the Euclidean sector or a non-minimum radius of the Lorentzian sector. Within the distributional approach, using Colombeau algebra, we obtain in general a non-vanishing expression for the energy-momentum tensor $S_{\mu\nu}$. In another words, one could require a matching along other sections corresponding to a non-maximum radius of the Euclidean sector (see Fig. 2) or a non-minimum radius of the Lorentzian sector (see Fig. 3). In these situations, energy-momentum of the change surface is non-vanishing. We consider this non-vanishing energy-momentum tensor as a possible source of primordial black holes production. Note that generalized uncertainty principle itself is a consequence of quantum fluctuation of background spacetime geometry. As has been indicated, in the spirit of path integral approach, Hartle-Hawking proposal is on the basis of quantum fluctuation of geometry itself. Therefore, one can argue that quantum fluctuation of geometry (which are considered in path integral) are the source of non-vanishing energy-momentum tensor (non-vanishing jump of the extrinsic curvature in the Darmois-Israel approach[19]) for change surface. This non-vanishing energy-momentum tensor can be considered as a source of BPHs production. Note that these two issues: geometric cosmological constant and non-vanishing energy-momentum tensor are related via the junction conditions

which can be derived from full field equations. In the next section we discuss the possible creation of primordial black hole from these two features of our model: geometric cosmological constant and non-vanishing energy-momentum tensor of the signature change hypersurface .

3 Signature Change and Primordial Black Hole Production

As has been indicated, black hole remnants (BHRs) are a natural candidate for cold dark matter since they are a form of weakly interacting massive particles (WIMPs). The possible source and abundance of BHRs are of interest. The most natural source is in primordial geometric fluctuations, which can be sufficiently large only in the Planck era, at about the Planck temperature. In previous sections we have provided a consistent framework for such geometric fluctuations in Planck era. Here, using analysis of Chen and Adler([4]and [17]) we address the possible PBHs production in the very early universe via the signature change. Rigorous derivations(for example analysis provided by Ohanian and Ruffini[20]) and also simple thermodynamic arguments(see for example [21]) imply that random fluctuations can produce a Boltzmann distribution of black holes, down to Planck mass, with a number density of $\sim \frac{1}{l_p^3}$. As Ohanian and Ruffini have shown[20], in one version of standard inflationary cosmology the scale factor can increase by a factor of about 10^{74} from the Planck era to the present, and since the number density of matter is related to the cube of this, we obtain a number density about $10^{-118}/m^3$. But the large scale density of dark matter is approximately equal to the critical density, that is, $\rho_{DM} \sim \rho_c \sim 2 \times 10^{-26} kg/m^3$. This results a BHR number density $\sim 10^{-18}/m^3$ which is impossible. In last sections we have provided a framework for an alternative signature changing cosmology which can support primordial black holes production. Using Hartle and Hawking no-boundary scenario for creation of the Universe, we have supposed that the universe was initially a truly chaotic quantum foam system without ordinary spacetime, and in particular without a time direction such that the signature was $(+, +, +, +)$. A fluctuation in the signature to $(-, +, +, +)$ can then produce a time direction and turn it into an exponentially expanding de Sitter space with heavy vacuum density(geometric cosmological constant) which can be equal to the Planck density. In this framework, at the beginning of the universe a thermal distribution of black holes can be produced

as has been pointed by Ohanian and Ruffini[20]. During the expansion, these thermal black holes can decay to form Planck size remnants and radiation. The presence of the black holes and radiation (photons, gravitons, etc.) changes the equation of state. Heavy vacuum(geometric cosmological constant) equation of state with $p = -\rho$, changes to a mixture of BHRs matter with $p = 0$ and radiation with $p = \rho/3$ (it is possible to have a very little residual vacuum energy also). This change of equation of state can change the scale factor from an exponential to a power-law form. In this situation apparently there is no horizon paradox[4,17].

Suppose that this transition occurs at a continuous energy density change. As a matching condition for creation of de Sitter phase, the scale factor should be continuous with continuous derivatives. Therefore, the scale factor can change according to following transformation

$$a(t) = e^{\frac{t}{t_p}} \Rightarrow a(t) = \left(\frac{e}{n}\right)^n \frac{t^n}{t_p^n} \quad (9)$$

at $t = nt_p$, where n should be between 1/2 for radiation and 2/3 for matter. The duration of exponential expansion will be quite short. One can estimate approximately the decrease in number density of BHRs from the beginning of time, $t = 0$, to the present time, $t = t_0$, as follows

$$\left[\frac{a(t_0)}{a(0)}\right]^3 \approx \left[\frac{t_0}{t_p}\right]^{3n} \quad (10)$$

Suppose that n is about 2/3(which is appropriate for matter), then the scale factor decreases by a factor of about 10^{41} and correspondingly, the density factor can decrease by a factor of about 10^{123} . This implies a present number density of black hole remnants of order $\rho_{BHR} \approx 10^{-18}m^{-3}$. This amount is reasonable. But the universe evolution contains a radiation dominated era also and we should consider it in our estimations. If we take into account a radiation dominated period of expansion, then n should be about 1/2 until the time of decoupling of matter and radiation, t_d , and the present density should be about $10^{10}/m^3$, which is very large to be reasonable. To overcome this difficulty, as has been pointed in [4,17], we have to consider an ad hoc period of inflation to obtain a reasonable density. By taking this strategy, if we extend the period of inflation from nt_p to ηt_p , followed by a period of radiation dominance to t_d , and then matter dominance to the present, we obtain[4]

$$\frac{a(t_0)}{a(0)} \approx \frac{e^\eta}{\sqrt{\eta}} \left(\frac{t_0^{2/3}}{t_p^{1/2} t_d^{1/6}} \right) \quad (11)$$

This gives a suitable PBH density if we chose the number of e -folding to be about ~ 27 ,

roughly half the number usually used in standard inflationary scenarios. The quantum field theoretical framework for actual mathematical formalism can be performed which is out of purpose of the present work. In which follows we discuss the important issue of possible detection of primordial black hole remnants.

4 Schemes for Possible Detection of Primordial Black Holes Remnants

It is by now widely accepted that dark matter (DM) constitutes a substantial fraction of the present critical energy density in the Universe. However, the nature of DM remains an open problem. There exist many DM candidates, most of them are non-baryonic weakly interacting massive particles (WIMPs), or WIMP-like particles. By far the DM candidates that have been more intensively studied are the lightest supersymmetric (SUSY) particles such as neutralinos or gravitinos, and the axions (as well as the axinos). There are additional particle physics inspired dark matter candidates. A candidate which is not as closely related to particle physics is the relics of primordial black holes (Micro Black Holes) which we have investigated its production. Certain inflation models naturally induce a large number of such a black holes. As a specific example, hybrid inflation can in principle yield the necessary abundance of primordial black hole remnants for them to be the primary source of dark matter. One of the major problems with these remnants is the possibility of their detection. As interactions with black hole remnants are purely gravitational, the cross section is extremely small, and direct observation of these remnants seems to be unlikely. One possible indirect signature may be associated with the cosmic gravitational wave background. Unlike photons, the gravitons radiated during evaporation would be instantly frozen. Since, according to our notion, the black hole evaporation would terminate when it reduces to a remnants, the graviton spectrum should have a cutoff at Planck mass. Such a cutoff would have by now been red-shifted to $\sim 10^{14} GeV$. Another possible gravitational wave-related signature may be the gravitational wave released during the gravitational collapse. The frequencies of such gravitational waves would by now be in the range of $\sim 10^7 - 10^8 Hz$. It would be interesting to investigate whether these signals are in principle observable. Another possible signature may be some imprints on the cosmic microwave background (CMB) fluctuations due to the thermodynamics of black hole remnants-CMB interactions. Possible production of such remnants in Large

Hadronic Collider (LHC) and also in ultrahigh energy cosmic ray (UHECR) air showers are under investigation. If we consider hybrid inflation as our primary cosmological model, there will be some observational constraints on hybrid inflation parameters. For example a simple calculation based on hybrid inflation suggests that the time it took for black holes to reduce to remnants is about $10^{-10} Sec$. Thus primordial black holes have been produced before baryogenesis and subsequent epochs in the standard cosmology[17]. The events that can potentially lead to black hole production are essentially high-energy scattering in particle colliders and UHECR. The next generation of particle colliders are expected to reach energies above $10 TeV$. For example, LHC is planned to reach a center-of-mass energy of $14TeV$. Therefore, if the fundamental Planck scale is of the order of few TeV , LHC would copiously produce black holes. Black hole production by cosmic rays has also been recently investigated by a number of authors(see for example[22,23] and references therein). Cosmogenic neutrinos[6] with energies above the Greisen- Zatsepin-Kuzmin (GZK) cutoff[7] are expected to create black holes in the terrestrial atmosphere. The thermal decay of the black hole produces air showers which could be observed. The cross sections of these events are two or more orders of magnitude larger than the cross sections of standard model processes. Therefore, black holes are created uniformly at all atmospheric depths with the most promising signal given by quasi-horizontal showers which maximize the likelihood of interaction. This allows black hole events to be distinguished from other standard model events. Detecting TeV black hole formation with UHECR detectors may be possible through the decay of τ -leptons generated by ν_τ 's that interact in the Earth or in mountain ranges close to the detectors. A secondary τ generated through the decay of a black hole has much less energy than the standard model τ secondary. In addition, black holes may produce multiple τ -leptons in their evaporation, a unique signature of TeV gravity. Standard model processes that generate multiple τ -leptons are highly unlikely, the detection of multiple τ 's in earth-skimming and mountain crossing neutrinos will be a smoking gun for black hole formation.

5 Summary

This paper provides a possible mechanism for primordial black hole production in early universe. The arguments presented here are based on the idea of signature change. It has been shown that junction condition for signature change can leads to a geometrical

cosmological constant. When we consider the existence of a minimal length scale on the order of Planck length, this geometric cosmological constant can be interpreted as a heavy vacuum density and this heavy vacuum density can be considered as a possible source of PBHs production. From another viewpoint, possible non-vanishing of energy-momentum tensor of the signature change hypersurface due to matching along non-maximum (or non-minimum) radius, can leads a possible source of PBHs production. These PBHs evaporates via Hawking process to reach eventually to a Planck size black hole remnants and these PBHRs can be interpreted as a source of dark matter. Possible experimental and observational schemes for detection of these primordial black hole remnants are discussed also.

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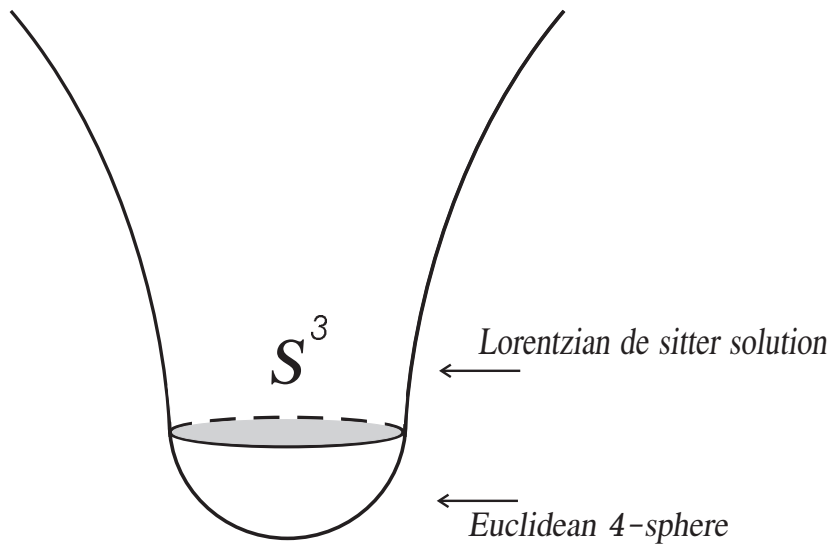


Figure 1: Creation of de Sitter Spacetime

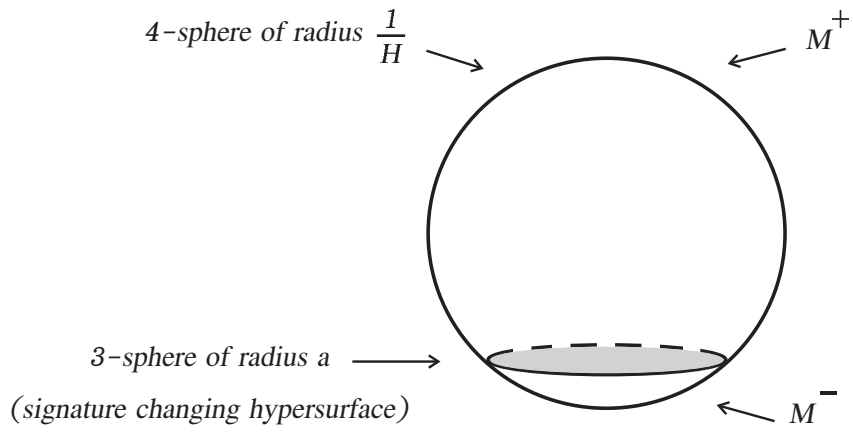


Figure 2: Matching on the signature changing hypersurface for the case of non-maximum radius in Euclidean sector

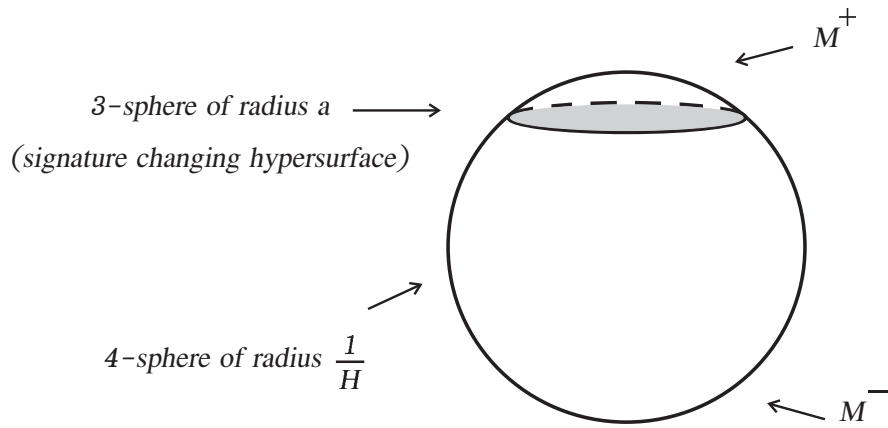


Figure 3: Matching on the Signature Changing Hypersurface for the case of non-maximum radius in Lorentzian sector.